RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [BATCH 2017-20]

B.A./B.Sc. SECOND SEMESTER (January – June) 2018

Mid-Semester Examination, March 2018

Date : 17/03/2018 — **MATH FOR ECONOMICS (General)**

Time : 11 am – 12 noon — Paper : II — Full Marks : 25

[Use a separate Answer Book for each group]

Group – A

(*Answer any two questions*) [2×5]

1. Let $D \subset \mathbb{R}$ and $f : D \to \mathbb{R}$ be a function on D. If any point $c \in D'$ then prove that f can have at most one limit at c, where D' is the derived set of D.

2. Prove that $\lim_{x \to 0} x \sin \frac{1}{x^2} = 0$.

3. Prove that a function f is defined on $\mathbb{R}$ by

   $f(x) = 1,\ x \in \mathbb{Q}$

   $= 0, x \in \mathbb{R} - \mathbb{Q}$

   is not continuous at any point on $\mathbb{R}$.

4. Prove that the function $f(x) = [x],\ 0 < x < 2$ has the jump discontinuity at x = 1.

Group – B

(*Answer any three questions*) [3×5]

5. a) Prove that the intersection of 2 subspaces of a vector space V over a field F is subspace of V. [3]

   b) Show that the set $S := \{f \in C_{\mathbb{R}}[0,1] \mid f(0) = 0\}$ is a subspace of the vector space $C_{\mathbb{R}}[0,1]$. [2]

   [where $C_{\mathbb{R}}[0,1]$ denotes the vector space of all real valued continuous functions on [0,1]]

6. Prove that a necessary & sufficient condition for a non-homogeneous system $AX = B$ to be consistent is rank A = rank $\bar{A}$, where $\bar{A}$ is the augmented matrix of the system. [5]

7. a) Find a basis & determine the dimension of the following subspace S of $\mathbb{R}_{2\times 2}$ where

   $$S := \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a + b = 0 \right\}.$$ [3]

   b) Prove that a set of vectors containing the null vector θ in a vector space V over a field F is linearly dependent. [2]

8. Determine the conditions for which the system

   $x + y + z = 1$

   $x + 2y - z = b$

   $5x + 7y + az = b^2$

   admits of (i) only one solution, (ii) no solution (iii) many solutions. [5]

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